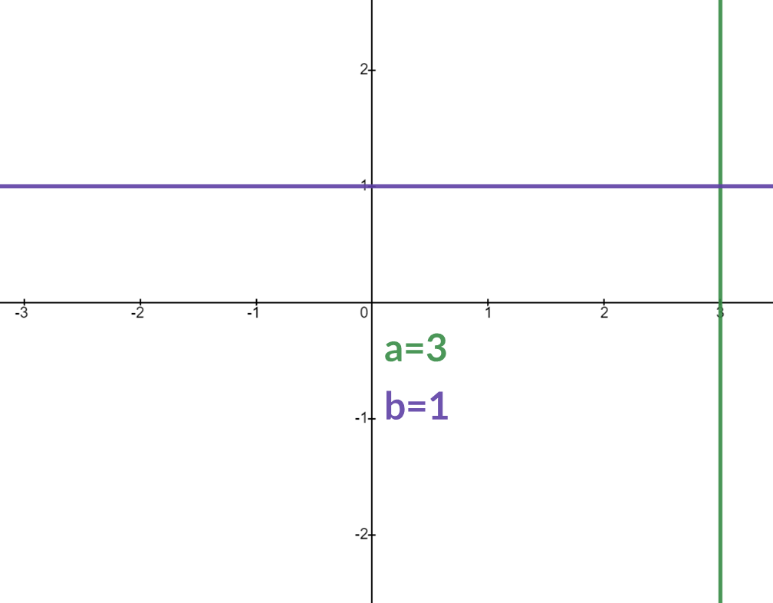
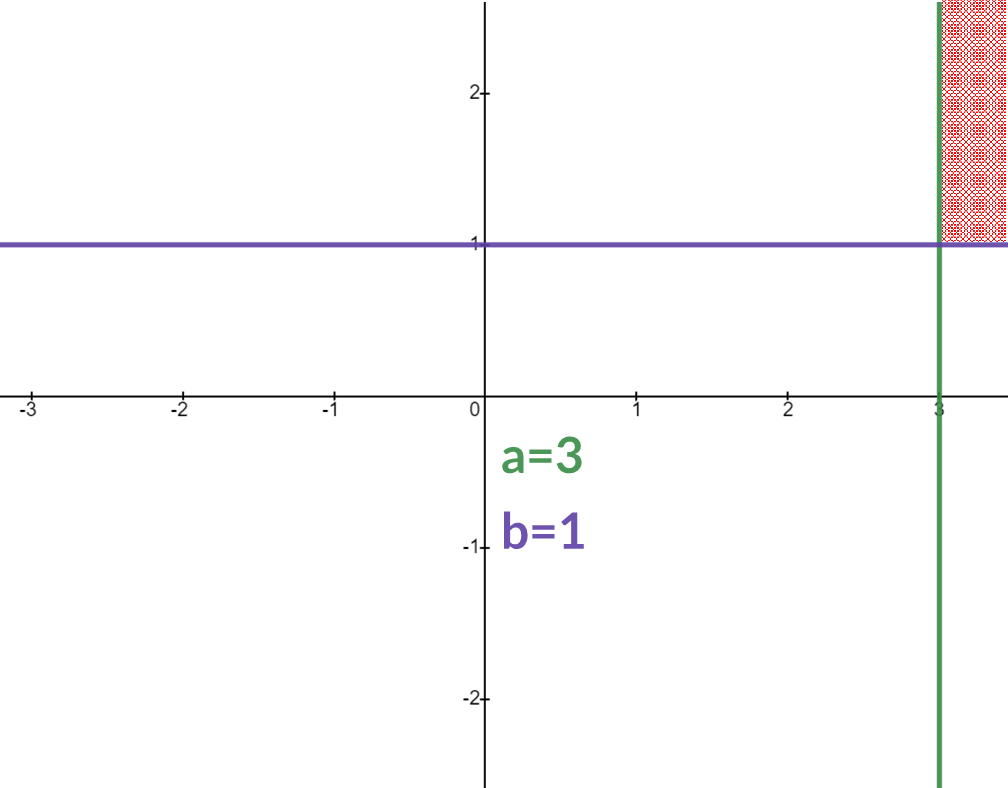
HW3 - Question 4 – Pac Learning

1. Suppose we have the classifier family: where iff . To find the VC dimension of this classifier family we need to understand it’s capabilities, we find the maximum amount of data points classifiers from can classify to every possible combination of classes. Let’s take as an example:

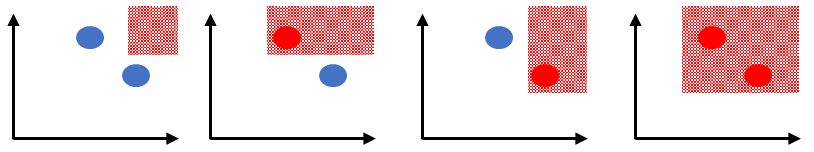


In this example the area where the classifier returns 1 is when, the area marked in red:



We will show that shatters a group of 2 samples into every possible combination of classifications but not for every group of 3 samples, hence the VC dimension would be 2.

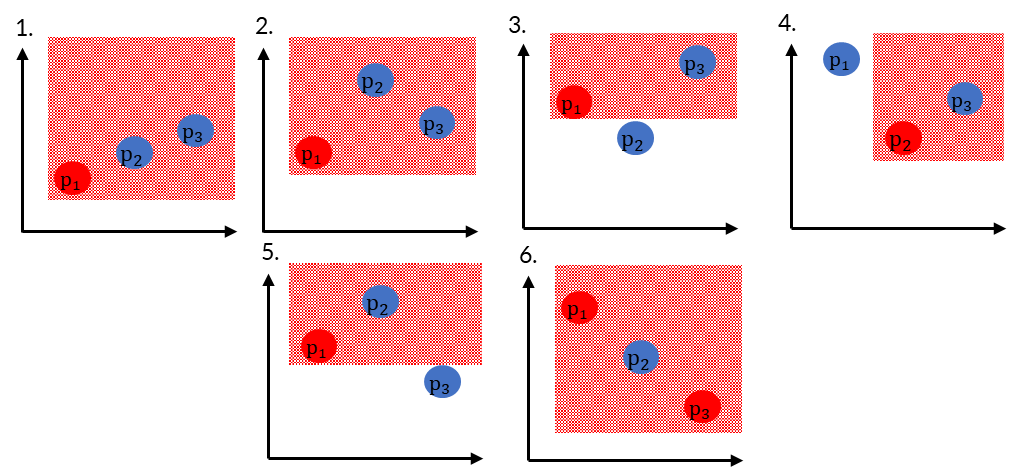
For every 2 samples that are on a straight line with a negative slope we can classify the 2 points into every possible combination of classifications using classifiers from .



The figure shows that the statement is true; The classifier is the area marked in red. The red and blue points are two types of classes.

shatters a group of 2 samples, hence the VC dimension of classifiers from is at least 2. We will now show that no 3 samples can be classified into every possible combination. We have 6 cases to consider, we will assume that :

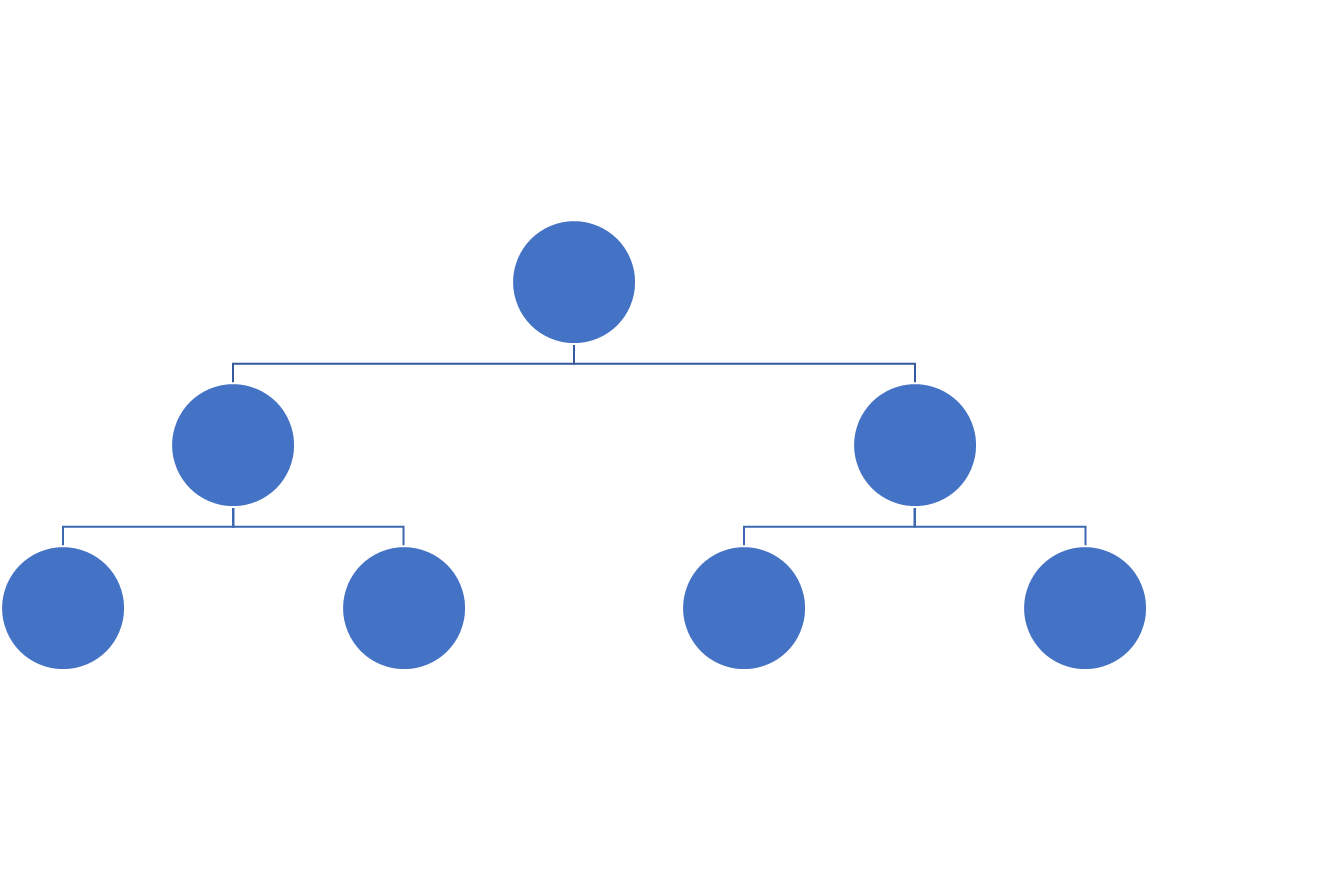
For each of these cases we will show that it is not possible to classify the points into every combination of classifications. The following scheme shows exactly that:



Each figure shows how we cannot shatter each case mentioned above.

It is impossible to shatter **every** 3 points with the classifier family , hence the VC dimension of is 2. ▪

1. Boolean variables, is calculated by: , i.e is a binary classifier. We try to learn using a decision tree of depth 2. Let’s define .



A decision tree of depth 2.

There is a total of 17 classifiers in , hence .